PM AND AM NOISE IN OSCILLATORS BASED ON WIDEBAND BJT AMPLIFIERS

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ABSTRACT

An approach to nonlinear quasistationary PM and AM noise calculation in BJT oscillators is presented. It is based on results of nonlinear analysis of PM and AM noise in BJT amplifiers. An influence of emitter current feedback resistor on PM noise of the oscillator is investigated. It is shown that there are optimal values of this resistor providing minima of wideband and 1/F PM noise. Results of nonlinear analysis are compared with “linear estimation” of PM noise.

1. INTRODUCTION

Since 1966 many engineers use an approach to estimation of PM noise power spectral density (PSD) in oscillators offered by D.Leeson - Ref. 1. This approach was developed for the oscillator model consisting of an amplifier and linear selective feedback circuit. PM noise of such oscillator was expressed in terms of PM noise of the amplifier and Q-factor of the feedback circuit. But to calculate PM noise of the amplifier one has to specify a signal source impedance and available power of the signal source. In some types of microwave amplifiers the signal source impedance is equal to standard value (50 Ohm) and available power can be found from experimental data or by simulation.

But in large variety of BJT oscillators a feedback circuit is connected directly to the BJT input. If one wants to use an approach similar to the Leeson’s one for such oscillators it is very important to define quite correctly the structures both an amplifier and a feedback circuit.

If the structure of the amplifier is defined, one can calculate PM noise in oscillators using results of PM noise analysis in BJT amplifiers that were obtained in the papers -Refs. 2-8. One can also investigate opportunities to reduce PM noise in oscillators using negative or complex feedback in the amplifier.

2. AN OSCILLATOR CIRCUIT DIAGRAM AND IT’S MODELS

Figure 1 shows the circuit diagram of a basic oscillator that is used to explain specific features of an approach to PM and AM noise calculation, developed in this paper. In this circuit diagram C_1, C_2, C_3, L, r are elements of Clapp oscillator feedback circuit, R_L and C_L – load resistance and capacitance, coupling the oscillator with R_L, L_{s1}, L_{s2}, r_{s1}, r_{s2} – inductances and resistances of the chokes, R_{B0}, R_{E0}, C_{B0}, C_{E0} – resistances and bypass capacitances of self-biasing circuits, Z_e – emitter feedback impedance.

A circuit model of this oscillator is shown Fig.2. In Fig. 2 the BJT is replaced by its charge control model – Ref. 6 with collector junction capacitance equal to zero. Power spectral densities of the noise sources shown in Fig.2 are given in – Refs. 6,7. This model describes rather well two BJT’s in common emitter-common base wideband amplifier – Ref. 5. Using emitter feedback impedance Z_e helps to increase the amplifier bandwidth. All elements of the feedback circuit are included in the two-port network that is described by its impedance matrix.

$$[Z]=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

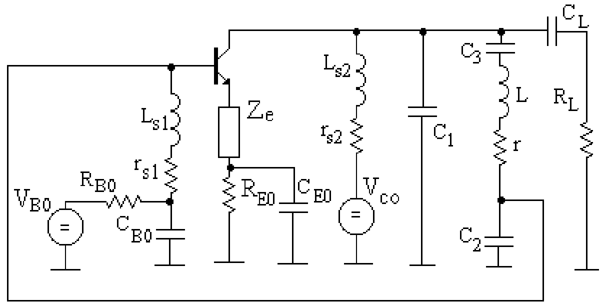


Fig.1. Circiut diagram of a Clapp oscillator

The oscillator circuit model Fig.2 consists of the amplifier and the feedback circuit, shown in Fig.3,a,b.

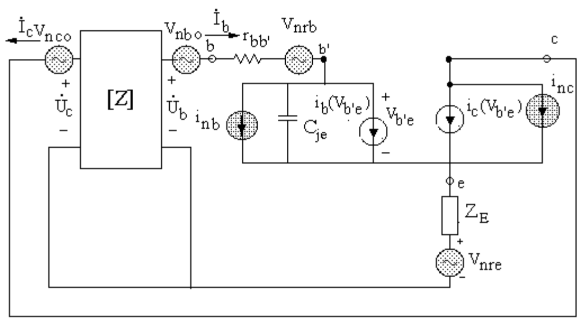


Fig.2. Circuit model of the feedback oscillator based on wideband BJT amplifier

3. BASIC EQUATIONS

Assuming that the feedback circuit half bandwidth is small by comparison to oscillation frequency $\omega_0 = 2\pi f_0$, we can take into consideration only first harmonics of open circuit voltage $V_{b01}(t)$ and collector current $i_{c1}(t)$:

$$v_{b01}(t)=\mathrm{Re}\left(\dot{V}_{b0}e^{j\omega_0 t}\right),\quad i_{c1}(t)=\mathrm{Re}\left(\dot{I}_c e^{j\omega_0 t}\right). \quad (1)$$

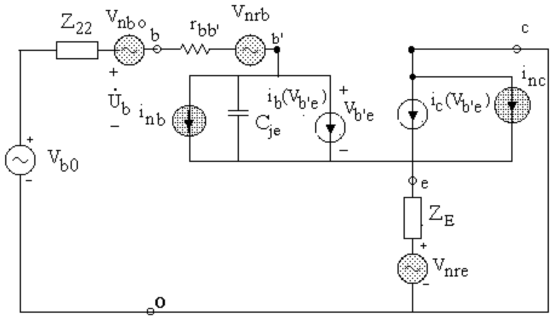
Using polyharmonic approach described in [6] we obtain a dependence of $\dot{I}_c$ on V_{b0} for the circuit Fig.3,a:

$$\dot{I}_c(V_{b0},\omega_0)=I_c(V_{b0},\omega_0)e^{j\varphi_v + j\varphi_{I_c}(V_{b0},\omega_0)}. \quad (2)$$

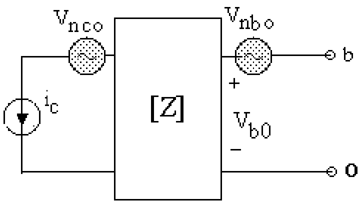
It is important to notice that the functions $I_{c1}(V_{b0},\omega_0)$ and $\varphi_{I_c}(V_{b0},\omega_0)$ have to be calculated taking into consideration an output impedance of the feedback circuit $Z_{22}(j\omega_0)$, and self-biasing resistance R_{E0} in Fig.1. On the other hand from

Fig.3,b $\dot{V}_{b0}$ can be expressed in terms of $Z_{21}(j\omega_0)$ and $\dot{I}_c(V_{b0},\omega_0)$

$$\dot{V}_{b0} = Z_{21}(j\omega_0)\dot{I}_c(V_{b0},\omega_0). \qquad (3)$$



a)



b)

Fig.3. Decomposition of the circuit model Fig.2:
a) an amplifier b) a positive feedback citcuit

An equation (3) describes steady-state operation of the oscillator. It has to be solved with respect to V_{b0} and ω_0 before PM and AM noise calculation. A complex equation (3) is equivalent to two equations

$$V_{b0} = |Z_{21}(j\omega_0)| I_c(V_{b0},\omega_0), \qquad (4)$$

$$\varphi_{Z_{21}(j\omega_0)} + \varphi_{Ic}(V_{b0},\omega_0) = 0. \qquad (5)$$

In (5) $\varphi_{Z_{21}(j\omega_0)}$ - is an argument of the mutual impedance $Z_{21}(j\omega_0)$.

Under influence of noise sources shown in Fig.3,a phase shift fluctuations $\phi_a(t)$ and fractional amplitude fluctuations $a_a(t)$ are introduced by the amplifier. That is why in the case when V_{b0}=const and ω_0=const at the output of the amplifier we have instead of (2):

$$\dot{I}_c(V_{b0},\omega_0) = I_c(V_{b0},\omega_0)[1 + a_a(t)]e^{j\varphi_v + j\varphi_{I_c} + j\phi_a(t)}. \qquad (6)$$

Algorithms and examples of $a_a(t)$ and $\phi_a(t)$ power spectral densities calculation are presented in – Refs.6,7.

In oscillator an influence of $a_a(t)$ and $\phi_a(t)$ gives rise to the feedback voltage amplitude and phase fluctuations so that instead of $\dot{V}_{b0}$ one has

$$\dot{V}_{bof}(t) = V_{bof}(t)e^{j\varphi_v + j\phi_v(t)} \qquad (7)$$

with

$$V_{b0f}(t) = V_{b0}(1 + a_v(t)) \qquad (8)$$

and instead of constant oscillation frequency ω_0 one has

$$\omega_{0f} = \omega_0 + \frac{d\phi_v}{dt}. \qquad (9)$$

For the modulation frequencies $\omega_m<\omega_r/2Q$ one can use quasi-static equations of the oscillator to find PSD's of $a_v(t)$ and $\phi_v(t)$. Replacing in (3) ω_0 by ω_{0f} and $\dot{V}_{bo}$ by $\dot{V}_{bof}(t)$ leads to the next equations

$$\begin{aligned} &V_{b0f}(t) = |Z_{21}(j\omega_{0f})| I_c(V_{b0f}(t), j\omega_{0f})[1 + a_a(t)] \\ &\varphi_{Z_{21}}(\omega_{0f}(t)) + \varphi_{I_C}(V_{bof}(t),\omega_{0f}(t)) + \phi_a(t) = 0 \end{aligned} \qquad (10)$$

As both amplitude and frequency fluctuations are very small one can linearize equations (10) in the vicinity of stationary values of V_{b0} and ω_0. Then taking (4), (5) into consideration one obtains the next equations

$$\left[1 - \frac{\partial I_c/\partial V_{b0}}{I_c/V_{b0}}\right] a_v(t) - \left[\frac{d\ln|Z_{21}|}{d\omega_0} + \frac{d\ln I_c}{d\omega_0}\right]\left(\frac{d\phi_v}{dt}\right) = a_a(t), \qquad (11)$$

$$\frac{\partial\varphi_{I_c}}{\partial V_{b0}} V_{b0} a_v(t) + \left[\frac{\partial\varphi_{Z_{21}}}{\partial\omega_0} + \frac{\partial\varphi_{Ic}}{\partial\omega_0}\right]\left(\frac{d\phi_v}{dt}\right) = -\phi_a(t). \qquad (12)$$

As a rule in low noise oscillators $|\varphi_{I_c}| < \pi/8$ and it is possible to neglect frequency – amplitude conversion term in (11) and amplitude-phase conversion term in (12). So for such oscillator one can use simplified equations

$$\left[1 - \frac{\partial I_c/\partial V_{b0}}{I_c/V_{b0}}\right] a_v(t) = a_a(t), \qquad (13)$$

$$\left[\frac{\partial\varphi_{Z_{21}}}{\partial\omega_0} + \frac{\partial\varphi_{Ic}}{\partial\omega_0}\right]\frac{d\phi_v}{dt} = -\phi_a(t). \qquad (14)$$

Let us introduce an oscillation amplitude sensitivity parameter

$$\lambda = \left[1 - \frac{\partial I_c/\partial V_{b0}}{I_c/V_{b0}}\right]^{-1} \qquad (15)$$

and phase-versus-frequency response characteristics (PFRC) slopes

$$\tau_Z = \frac{\partial\varphi_{Z_{21}}}{\partial\omega_0}, \qquad \tau_I = \frac{\partial\varphi_{I_c}}{\partial\omega_0}, \qquad (16)$$

and present the amplifier AM and PM noises as sums of wideband components $a_{aw}(t)$ and $\phi_{aw}(t)$ and 1/F type components $a_{af}(t)$, $\phi_{af}(t)$

$$a_a(t) = a_{aw}(t) + a_{af}(t),\ \phi_a(t) = \phi_{aw}(t) + \phi_{af}(t). \qquad (17)$$

Then from (13), (14), one expresses the power spectral densities (PSD) $S_{av}(\omega)$, $S_{\phi v}(\omega)$ of $a_v(t)$ and $\phi_v(t)$ in terms of PSD's $S_{a,aw}(\omega)$, $S_{a,af}(\omega)$, $S_{\phi,aw}(\omega)$, $S_{\phi,af}(\omega)$ of $a_{aw}(t)$, $a_{af}(t)$, $\phi_{aw}(t)$, $\phi_{af}(t)$.

$$S_{av}(\omega) = \lambda^2[S_{a,aw}(\omega) + S_{a,af}(\omega)], \qquad (18)$$

$$S_{\phi v}(\omega) = \frac{1}{\omega^2(\tau_Z + \tau_I)^2}[S_{\phi,aw}(\omega) + S_{\phi,af}(\omega)], \qquad (19)$$

In these equations ω - Fourier frequency.

One can see from (18), (19) that using algorithms of AM and PM noises calculation in BJT amplifiers presented in - Refs.5,7 the AM and PM noises in oscillator can be calculated.

4. PM NOISE IN A SINGLE RESONATOR FEEDBACK OSCILLATOR

In more detail we consider PM noise for a single resonator feedback circuit shown in Fig.1. In this case

$$Z_{21}(j\omega_0)=\frac{R}{1+j2(\omega_0-\omega_r)Q/\omega_r}, \qquad (20)$$

where ω_r - a resonant frequency, Q – quality factor, $R=Z_{21}(j\omega_r)$ – a resonant mutual impedance, and

$$Z_{22}(j\omega_0)=jX_2+k_f Z_{21}(j\omega_0) \qquad (21)$$

Where $X_2=-1/\omega_r C_2$ and $k_f = C_1/C_2$.

In accordance with – Refs. 6 PSD of the wideband PM noise of the amplifier shown in Fig.3,a can be presented by formula

$$S_{\phi,aw}(\omega)=F_\phi\frac{k_B T}{P_{av}}, \qquad (22)$$

where

$$P_{av}=V_{b0}^2/8\,\mathrm{Re}\,Z_{22}(j\omega_0) \qquad (23)$$

- available power of the signal source shown in Fig.3,a , k_B – Boltzmann constant, T – absolute temperature, F_ϕ - “phase” noise factor, that is calculated with signal source impedance $Z_{22}(j\omega_0)$ taking into consideration nonlinear effects – Refs.6. It is not difficult to prove that for single resonator circuits

$$P_{av}=P_0/4\,. \qquad (25)$$

Where P_0 – a power, delivered in the resonator by the current source $i_c(t)$.

From (16), (20) it follows that

$$\tau_Z=\frac{1}{1+4(\omega_0-\omega_r)^2Q^2/\omega_r^2}\frac{2Q}{\omega_r}\quad . \qquad (26)$$

If $\left|\varphi_{I_c}\right|<<1$ then

$$\left|\omega_0-\omega_r\right|<<\omega_r/2Q \qquad (27)$$

and

$$\tau_Z=\frac{2Q}{\omega_r} \qquad (28)$$

as it is assumed in –Ref.1.

The PFRC slope τ_I accounts for influence of the BJT input impedance on quality factor of the resonator. It is usually negative. For single resonator feedback circuit it can be taken into account if one replaces Q by “loaded” quality factor Q_L, so that

$$\tau_Z+\tau_I=\frac{2Q_L}{\omega_r}\quad . \qquad (29)$$

In our approach τ_I was calculated numerically from (16). It gives an opportunity to find Q_L, using (28).

To calculate 1/F PM noise PSD $S_{\phi,af}(\omega)$ we express it in terms of coefficient T_φ that accounts for transformation of relative fluctuations of recombination component of the base current $\mu(t)$ in fluctuations of the phase shift $\phi_{af}(t)$:

$$S_{\phi,af}(\omega)=T_\varphi^2 S_\mu(\omega)\,. \qquad (30)$$

where $S\mu(\omega)$ – PSD of $\mu(t)$. A method of T_φ calculation was described in – Ref. 7.

From (19), (22), (29) we obtain

$$S_{\phi v}(\omega)=\frac{1}{\omega^2(\tau_Z+\tau_I)^2}\left(F_\phi\frac{k_B T}{P_{av}}+T_\varphi^2 S_\mu(\omega_m)\left(\frac{\omega_m}{\omega}\right)\right), \qquad (31)$$

where ω_m – Fourier frequency at which PSD $S\mu(\omega)$ was measured. It is important to notice that result of T_φ calculation depends on the value of $Z_{22}(j\omega_0)$. This dependence is especially important near the points of 1/F PM noise compensation – Ref. 8.

For a single resonator oscillator with small difference between ω_0 and ω_r (27) it is possible to use (28) and (29) replace (32) by formula

$$S_{\phi v}(\omega)=\frac{(\omega_r/2Q_L)^2}{\omega^2}\left(F_\phi\frac{k_B T}{P_{av}}+T_\varphi^2 S_\mu(\omega_m)\left(\frac{\omega_m}{\omega}\right)\right) \qquad (32)$$

that is similar to the one obtained in – Ref.1.

5. AN INFLUENCE OF EMITTER CURRENT FEEDBACK ON PM NOISE

Let us consider an influence of emitter feedback resistor on PM noise of the oscillator shown in Fig.1.

The BJT that is used in this oscillator has the same parameters as the one considered in the papers – Refs. 5-8: $r_{bb'}$ =30 Ohm, β=50, $(\omega_T/2\pi)$=500 MHz, C_{je} = 5 pF. But in this case we put C_{jc} = 0. We assume also that at the point of oscillations self-exiting the BJT has collector current I_{C0} = 5 mA. At this operating point the BJT incremental model parameters are: g_m = 0.2 A/V, $r_\pi=\beta/g_m$=250 Ohm, $C_\pi=(g_m/\omega_T)+C_{je}$ = 64 pF.

The oscillator under consideration has $(\omega_0/2\pi)$=20 MHz, $Z_{11}(j\omega_0)=Z_{11}$=1 kOhm. In this oscillator we change $Z_e=R_e$ from 0 to 500 Ohm . For each value of R_e parameters of the feedback circuit are chosen to support constant values of ω_0, Q=50, I_{C0} = 5 mA, $I_{C1}=I_{C0}*(\pi/2)$=7,85 mA (it corresponds to loop gain compression ratio equal to 2). We also supported

$$\frac{R_{E0}}{g_m^{-1}+(R_{B0}/\beta)+R_e}=10,$$

where R_{B0} = 2,5 kOhm. The power P_0, delivered by the BJT collector current in the resonator was equal to 30,8 mW, and in accordance with (25) P_{av}=7,52 mW. Under these conditions we found the values of $Z_{22}(j\omega_0)$ corresponding to each value of R_e and calculated $\left(\frac{Q}{Q_L}\right)^2F_\phi$ and $\left(\frac{Q}{Q_L}\right)^2T_\phi^2$ as functions of (g_mR_e). Results of these calculations are shown in Fig.4,5 (solid lines). One can see that both functions have minima near point g_mR_e = 30. Reduction of 1/F PM noise due to R_e optimization is more significant than reduction of wideband PM noise.

To estimate an error of linear approach to PM noise calculation we found $Q_{L,lin}$, noise factor F and transformation coefficient $T_{\phi,lin}^2$ supposing small signal BJT operation at the point I_{C0} = 5 mA with signal source having the same frequency ω_0 and impedance $Z_{22}(j\omega_0)$. Then using formula

$$\left[S_{\phi v}(\omega)\right]_{lin}=\frac{(\omega_r/2Q_{L,lin})^2}{\omega^2}\left(F\frac{k_B T}{P_{av}}+T_{\varphi,lin}^2 S_\mu(\omega_m)\left(\frac{\omega_m}{\omega}\right)\right),$$

we calculated linear estimation of PM noise in the oscillator under consideration.

Results of these calculations are shown in Fig. 4,5 by dotted lines. It is interesting to notice that for wideband noise “linear estimation” is less than result of nonlinear calculation and it is vice versa for 1/F PM noise. The difference between results of linear estimation and nonlinear analysis becomes more when loop gain compression ration increases.

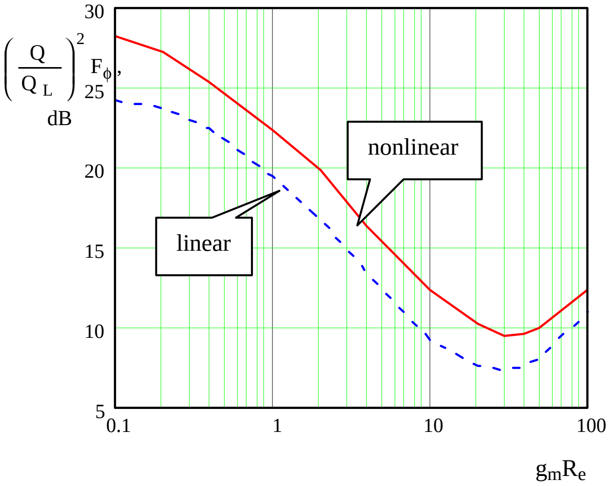


Fig.4. Wideband PM noise characteristics as functions of emitter feedback parameter

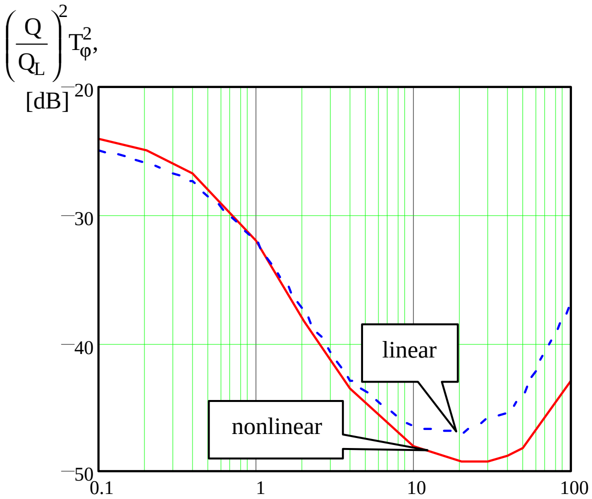


Fig.5. 1/F PM noise characteristics as functions of emitter feedback parameters

6. CONCLUSION

An approach developed in this paper was here applied to rather low frequency oscillator. It helped to obtain optimal values of R_e. This approach can be used for systematic investigation and comparison of PM and AM noise in oscillators that have higher frequencies, complex emitter feedback, and other types of feedback resonators, to reveal new opportunities to decrease PM and AM noise.

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